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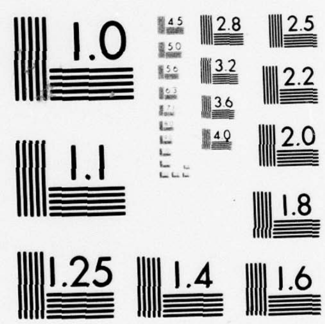


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# NON-NEUTRAL FIELD-ALIGNED CURRENT SHEET AND AURORAL ELECTRIC FIELD

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Final Report
15 May 1978 - 31 December 1978

Date of Report
February, 1979



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AIR FORCE SYSTEMS COMMAND
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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

| 19 REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|---|---|---|
| 1. REPORT NUMBER<br>18 AFGL-TR-79-0066 | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER<br>9 |
| 4. TITLE (and Subtitle)<br>6 NON-NEUTRAL FIELD-ALIGNED CURRENT SHEET AND AURORAL ELECTRIC FIELD. | | 5. TYPE OF REPORT & PERIOD COVERED<br>Final Report.<br>15 May 1978 — 31 Dec 1978 |
| | | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s)<br>10 R.K. Albano,<br>L.C. Lee<br>J.R. Kan | | 8. CONTRACT OR GRANT NUMBER(s)<br>15 F19628-78-C-0128 |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>Geophysical Institute<br>University of Alaska<br>Fairbanks, Alaska 99701 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS<br>61102F<br>16 2311G2AB 17 G2 |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>Air Force Geophysics Laboratory<br>Hanscom AFB, Massachusetts 01731<br>Monitor/Michael Smiddy/PHR | | 12. REPORT DATE<br>11 February 1979 |
| | | 13. NUMBER OF PAGES<br>19 |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)<br>12 21p. | | 15. SECURITY CLASS. (of this report)<br>Unclassified |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Aurora, Electric field, current sheet

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Equilibrium configurations of the non-neutral field-aligned electron current sheet in a Maxwellian plasma are obtained from solutions of the time-independent Vlasov-Maxwell equations. This is the first field-aligned non-neutral current sheet model in a hot plasma in which the electrons are allowed to be nonadiabatic. The current in this model has a perpendicular as well as a parallel component to the external magnetic

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field. The electric field of the current sheet is pointing normally toward the midplane of the sheet. This zeroth-order perpendicular electric field is identified as the primary electric field which gives rise to a few keV potential drop along auroral field lines.

## 1. Introduction

The zeroth-order electric field associated with a field-aligned electron current sheet is of fundamental importance in the formation of discrete auroras as has been proposed by Kan and Akasofu (1979). This electric field is perpendicular and pointing toward the midplane of the sheet. Intense electric fields of this type have been observed on auroral field lines at altitudes of a few earth radii ($R_E$) by Wescott et al. (1976) and Mozer et al. (1977).

Recently Kan, Lee and Akasofu (1979) showed that the perpendicular electric field of the field-aligned current sheet develops into a V-shaped potential structure due to the high conductivity in the ionosphere. As a result, current-carrying electrons are accelerated downward along magnetic field lines by the parallel electric field in the V-potential region and produce discrete auroras.

The purpose of this paper is to show the existence of non-neutral electron current sheets for adiabatic as well as nonadiabatic electrons in a Maxwellian plasma and to relate the characteristics of the current sheet to the thin auroral arcs of a few hundred meters in latitudinal thickness (Maggs and Davis, 1968). Non-neutral electron current sheet equilibrium in a monoenergetic plasma has been demonstrated under the assumption that the current is exactly parallel to the external field lines (Kan and Akasofu, 1979). It will be shown in this paper that the confinement of the non-neutral electron current sheet in a finite temperature Maxwellian plasma depends on the diamagnetic current perpendicular to the external magnetic field rather than the parallel current.

## 2. Basic Equations

Equilibrium configurations of exact charge-neutral current sheets in a Maxwellian plasma have been studied by Harris (1962), Nicholson (1963), and Kan (1972). However, a current sheet need not be exactly neutral, especially when the current is carried primarily by the electrons.

A one-dimensional model of the non-neutral current sheet is formulated on the basis of the Vlasov-Maxwell equations. The current sheet is assumed parallel to the y-z plane and centered at $x = 0$. The external magnetic field lies in the positive z-direction. All physical quantities may depend on x only.

The Vlasov equation is satisfied by any function of the constants of motion (Longmire, 1963). However, there are constraints on the function and its argument imposed by the prescribed boundary conditions of a given current sheet configuration. In this one-dimensional geometry the constants of motion for the s-th species are

$$H_s = \frac{1}{2} m_s (v_x^2 + v_y^2 + v_z^2) + q_s \phi(x)$$

$$P_{ys} = m_s v_y + (q_s/c) A_y(x) \qquad (1)$$

$$P_{zs} = m_s v_z + (q_s/c) A_z(x)$$

The distribution function is assumed Maxwellian and can be written as

$$f_s(x, \chi) = K_s \exp(-\varepsilon_s/\theta_s) \qquad (2)$$

where the argument of the exponential function, $\varepsilon_s$, is an energy in terms of the constants of the motion, $\theta_s$ is the thermal energy, and $K_s$ is a normalization constant. In general, $\varepsilon_s$ can be any linear combination of $H_s$, $P_{ys}$, $P_{yz}$, $P_{ys}^2$ and $P_{zs}^2$. However, a particular combination will be chosen such that the solutions satisfy the desired boundary conditions.

<u>Nonadiabatic-electron current sheet</u>:

The field-aligned current velocity V is assumed to be constant and in the positive z direction. A diamagnetic current is allowed to flow in the y direction. Under these assumptions, the $\varepsilon$ quantity for the current-carrying electrons is chosen to be

$$\varepsilon_c = H_c - VP_{zc} + \frac{1}{2} mV^2 + bP_{yc}^2/2m \tag{3}$$

where b is a dimensionless constant pertaining to the diamagnetic current and m is the electron mass. Upon substituting (3) into (2) the distribution function of the current-carrying electrons can be written as

$$f_c = \frac{(1 + b)^{1/2}}{\pi^{3/2} v_{oc}^3} n_c \exp\left\{\left[v_x^2 + (1 + b)\left(v_y + \frac{b}{1 + b}\frac{e}{mc} A_y\right)^2 + (v_z - V)^2\right]/v_{oc}^2\right\} \tag{4}$$

where

$$n_c = N_c \exp\left\{\left[\frac{Ve}{c} A_z + e(\phi - \phi_1) - \frac{b}{1 + b}\frac{e^2}{2 mc^2} A_y^2\right]/\theta_c\right\} \tag{5}$$

and $\phi_1 = \phi$ (x = 0).

The background electrons and background ions are assumed stationary. Density gradients are created in the background electrons and ions by the electric field of the current sheet. These density gradients produce no current. The distribution function of the background electrons may be written as

$$f_e = \frac{1}{\pi^{3/2} v_{oe}^3} n_e \exp\left[-(v_x^2 + v_y^2 + v_z^2)/v_{oe}^2\right] \quad (6)$$

where

$$n_e = N_e \exp(e\phi/\theta_e) \quad (7)$$

The background ion distribution function can be written similar to (6) as

$$f_i = \frac{1}{\pi^{3/2} v_{oi}^3} n_i \exp\left[-(v_x^2 + v_y^2 + v_z^2)/v_{oi}^2\right] \quad (8)$$

where

$$n_i = N_i \exp(-e\phi/\theta_i) \quad (9)$$

It is convenient to introduce the following normalized quantities

$$\theta_s^* = \theta_s/\theta_o,\ n_s^* = n_s/N_o,\ N_s^* = N_s/N_o$$

$$x^* = x/\lambda_D,\ V^* = V/v_o,\ E^* = E/E_o \quad (10)$$

$$\underset{\sim}{B}^* = \underset{\sim}{B}/B_o,\ \underset{\sim}{A}^* = \underset{\sim}{A}/B_o\lambda_D,\ \phi^* = \phi/\phi_o$$

where $\theta_o$ is a unit thermal energy, $N_o$ is a unit number density, $B_o$ is the external magnetic field, $\lambda_D = (\theta_o/4\pi N_o e^2)^{1/2}$ is the plasma Debye length, $\phi_o = \theta_o/e$, $E_o = \phi_o/\lambda_D$ and $v_o = (2\theta_o/m)^{\frac{1}{2}}$.

The time-independent Maxwell equations, with the aid of the distribution functions in (4), (6) and (8) can be written in dimensionless form as

$$\frac{d^2\phi^*}{dx^{*2}} = n_c^* + n_e^* - n_i^* \tag{11}$$

$$\frac{d^2A_y^*}{dx^{*2}} = \gamma\alpha^2 \; A_y^* \; n_c^* \tag{12}$$

$$\frac{d^2A_z^*}{dx^{*2}} = \sqrt{\beta\gamma} \; V^* \; n_c^* \tag{13}$$

where

$$n_c^* = N_c^* \exp \; (-\gamma V^* A_z^*/B + \phi^* - \phi_1^* - \gamma\alpha^2 A_y^{*2}/B \tag{14}$$

$$n_e^* = N_e^* \exp \; (\phi^*/\theta_e^*) \tag{15}$$

$$n_i^* = N_i \exp \; (-\phi^*/\theta_i^*) \tag{16}$$

The parameter $\beta = 8\pi N_o \theta_o/B_o^2$, $\gamma = \theta_o/mc^2$ and $\alpha^2 = b/(1 + b)$ where b is a parameter introduced in (3). Note that $N_c^*$ is the number density of the current-carrying electrons at $x^* = 0$, and $N_e^* = N_i^* = N^*$ is the number density of the background plasma outside the current sheet.

For the auroral field-aligned current sheet $\beta \sim 10^{-4}$ and $\gamma \sim 10^{-2}$, the right hand side of (12) and (13) can be set to zero. One obtains

$$A_y^* = B^* x^*, \; A_z^* = 0 \tag{17}$$

The number density in (14) can be rewritten as

$$n_c^* = N_c^* \exp \; (\phi^* - \phi_1^*)/\theta_c^* - x^{*2}/\ell^{*2} \tag{18}$$

where $\ell^* = (\beta\theta_c^*/\gamma\alpha^2 B^{*2})^{1/2}$ is the normalized half-width of the nonadiabatic-electron current sheet.

Equation (11) can be solved numerically subject to the boundary conditions

$$\phi^* \; (x^* = 0) = \phi_1^* = -\phi_m^*$$

$$\frac{d\phi^*}{dx^*} \; (x^* = 0) = 0$$

$$\lim_{|x^*| \to \infty} \phi(x^*) = 0 \tag{19}$$

$$\lim_{|x^*| \to \infty} \frac{d\phi^*}{dx^*} (x^*) = 0$$

where $\phi_m^*$ is a constant potential to be determined. There are six parameters in our problem $N_e^* = N_i^* = N^*$; $N_c^*$; $\theta_e^*$; $\theta_i^*$; $\theta_c^*$; and $\ell^*$. For a given set of parameters we must find the value of $\phi_m^*$. This is accomplished by integrating equation (11) starting from $x^* = 0$ using a trial value for $\phi_m^*$. The trial value for $\phi_m^*$ is varied until the boundary conditions at large $x^*$ in (19) are satisfied.

Adiabatic-electron current sheet:

Adiabatic electrons follow the $\underset{\sim}{E} \times \underset{\sim}{B}$ drift motion in the y direction. This motion cannot affect the number density which can only depend on x. Hence, the background electrons can be assumed uniform, i.e.

$$n_e^* = N_e^* \tag{20}$$

Similarly, the number density of the current-carrying electrons is independent of the electric field and is determined externally by the current source. Hence, it can be assumed to have a Gaussian-shaped profile, i.e.,

$$n_c^* = N_c^* \exp\left(-x^{*2}/\ell^{*2}\right) \tag{21}$$

where $\ell^*$ is the half-width of the density profile. In this case, the thickness of the current sheet $2\ell^*$ is determined by the current source as long as it is much greater than the electron gyrodiameter. The ion number density is still given by (16) as long as the ions are not completely magnetized.

## 3. Numerical Results

In this section we show the numerical results pertaining to the two types of current sheets formulated in the previous section. In Section 4 we apply these results to discrete auroral arcs characterized by a specific choice of $N_o$ and $\theta_o$.

For all examples in this section we fix $\theta_e^* = \theta_c^* = 0.1$; $\theta_i^* = 1.0$; and $N_i^* = N_e^* = 1.0$ for the plasma parameters along auroral field lines. The value of $\phi_m^*$ is then determined by our choice of $\ell^*$ and $N_c^*$. We consider current sheets where the half-width is on the order of the Debye length, i.e., $\ell \sim \lambda_D$. Under the quasi-neutrality approximation, equation (11) evaluated at $^* = 0$ can be written as $\phi_m^* \sim \theta_i^* \ln N_c^*/N_i^*$, which shows that the electric potential is on the order of the ion thermal energy. For a given $\ell^*$ we choose $N_c^*$ such that $\phi_m^*$ is on the order of $\theta_i^*$.

<u>Nonadiabatic-electron current sheet</u>:

To illustrate nonadiabatic-electron current sheet solutions two examples are considered. The first example is present in Figure 1 with $\ell^* = 0.5$ and $N_c^* = 4.06$. Figure 1a shows the number densities of the ions and electrons. The width of the electron density profile is on the order of the Debye length. Note that the current-carrying electrons are confined within $x^* = 1.5$ while the background electrons are pushed out of the current sheet by the electric potential $\phi^*$. This behavior is the cause of the dip in the electron density profile near $x^* = 1.8$. The ions are attracted to the electron current sheet and partially shield the electric field. As shown in Figure 1b, the depth of the potential profile is on the order of the ion thermal energy ($m = 1.2\ \theta_i$). The maximum electric field is $E_m^* = 0.8$.

The second example is shown in Figure 2 with $\ell^* = 0.2$ and $N_c^* = 7.7$. The depth of the potential well $\phi_m^*$ is about 1.2. In this case, the current-carrying electrons have a higher number density and the current sheet is confined to a smaller region when compared to the example shown in Figure 1. As a result, the current sheet supports a larger electric field, $E_m^* = 1.4$. Note that the dip in the electron density profile in Figure 2a is deeper than the dip in Figure 1a.

Adiabatic-electron current sheet:

Figure 3 shows an adiabatic-electron current sheet solution with $\ell^* = 1.0$ and $N_c^* = 3.3$. The choice of $N_c^* = 3.3$ gives $\phi_m^* = 1.2$. The width of the electron density profile is on the order of the Debye length. The maximum electric field is $E_m^* = 0.6$. There is no dip in the electron density profile because the electron distribution functions are not affected by the formation of the electric potential.

## 4. Application to Discrete Auroral Arcs

The numerical solutions we have obtained are valid for any choice of $N_o$ and $\theta_o$.

Consider an auroral field-aligned electron current sheet at 1 $R_E$ altitude where $N_o = 1\ cm^{-3}$ and $\theta_o = 5$ kev are the particle number density and the ion thermal energy, respectively. The magnetic field at 1 $R_E$ is about 0.06 Gauss. The Debye length is $\lambda_D \sim 500$ m and the electron gyroradius is $\rho_e \sim 12$ m in the absence of the electric field. In this case, electron motions are adiabatic. Hence, we can apply the solution shown in Figure 3 to obtain $\theta_c = \theta_e = 0.5$ kev, $\theta_i = 5$ kev, $N_e = N_i = 1\ cm^{-3}$, and $N_c = 3.3\ cm^{-3}$. The half-width of the current layer is ~ 500 m. The depth of the potential profile is ~ 6 kev and the maximum electric field is ~ 6 V/m.

As a second example, we consider a current sheet at 2 $R_E$ altitude in which $N_o = 0.3\ cm^{-3}$, $\theta_o = 5$ kev are the typical particle density and the typical ion thermal energy, respectively. The magnetic field at 2 $R_E$ is $\sim$ 0.02 Gauss. In this case, $\lambda_D \sim$ 860 m and $\rho_e \sim$ 45 m in the absence of the electric field. It can be shown from numerical calculation of particle trajectories that the electrons are nonadiabatic if the half-width $\ell$ is less than $\sim$ 200 m. The nonadiabatic-electron current sheet solution shown in Figure 2 is applicable since $\ell = 0.2\ \lambda_D \sim$ 170 m. For the solution in Figure 2 we have $N_e = N_i = 0.3\ cm^{-3}$; $N_c = 2.3\ cm^{-3}$; $\theta_e = \theta_c = 0.5$ kev; and $\theta_i = 5$ kev. The depth of the potential profile is $\sim$ 6 kev and the maximum electric field in the current sheet is $\sim$ 8 V/m.

## 5. Conclusion

We have shown the existence of non-neutral field-aligned electron current sheets in a Maxwellian plasma for adiabatic as well as nonadiabatic electrons. The electric fields are confined within the current sheet due to the shielding effect of the background ions. If the current-carrying electrons are nonadiabatic, the electron number density profile depends on the electrostatic potential. On the other hand, if the electrons are adiabatic, the electron number density profile is determined externally by the current source.

The results show that the total potential energy in the non-neutral current sheet is on the order of the ion thermal energy which ranges from 1 to 10 keV in the plasma sheet. The model gives a maximum electric field ranging from 1 to 10 V/m. Due to the short-circuiting effect of the conductive ionosphere, the total potential of the current sheet will be forced to be distributed along magnetic field lines as has been shown by Kan *et al*. (1979). This energy is comparable to the energy of the precipitating auroral electrons.

Acknowledgments: This work was supported in part by the Air Force Geophysics Laboratory, Air Force Systems Command under Contract F19628-78-C-0128 to the University of Alaska.

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## Figure Captions

Figure 1 A nonadiabatic-electron current sheet solution for $\ell/\lambda_D = 0.5$.

(a) The dimensionless electron and ion number density profiles as a function of the dimensionless distance, $x/\lambda_D$, from the center of the current sheet.

(b) The electric potential and perpendicular electric field profiles as a function of $x/\lambda_D$. For the conditions at $\sim 2\ R_E$ altitude on auroral field lines: $N_o = 0.3\ cm^{-3}$ and $\theta_o = 5$ keV giving $\lambda_D = 860$ m; $\phi_o = 5$ kV; and $E_o = 5.5$ V/m.

Figure 2 A nonadiabatic-electron current sheet for $\ell/\lambda_D = 0.2$.

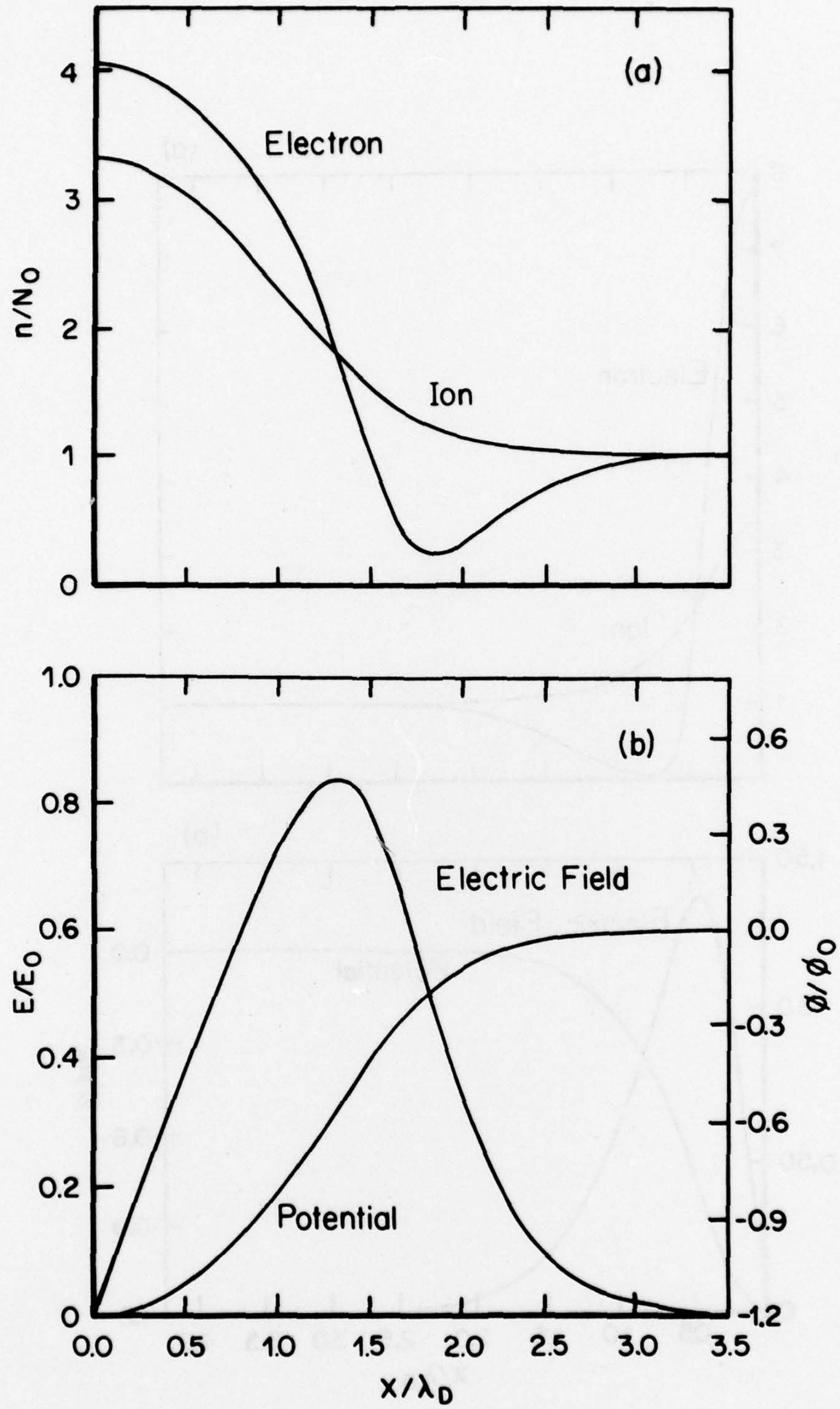
(a) The dimensionless ion and electron number density profiles as a function of $x/\lambda_D$. The number density of current carrying electrons is greater than for Figure 1, and the current carrying electrons are confined to a smaller region.

(b) The electric potential and perpendicular electric field as a function of $x/\lambda_D$. The electric field supported by the current sheet is larger than in Figure 1. For the condition at $\sim 2\ R_E$ altitude on auroral field lines: $N_o = 0.3\ cm^{-3}$ and $\theta_o = 5$ keV giving $\lambda_D = 860$ m; $\phi_o = 5$ kv; and $E_o = 5.5$ V/m.

Figure 3 An adiabatic-electron current sheet solution for $\ell/\lambda_D = 1.0$.

(a) The dimensionless ion and electron number density profiles as a function of $x/\lambda_D$.

(b) The dimensionless electric potential and perpendicular electric field as a function of $x/\lambda_D$. For the conditions at $\sim 1\ R_E$ altitude on auroral field lines: $N_o = 1\ cm^{-3}$ and $\theta_o = 5$ keV giving $\lambda_D = 500$ m; $\phi_o = 5$ kV; and $E_o = 9.5$ V/m.



(a)
Electron
Ion
n/N0
0
1
2
3
4
(b)
Electric Field
Potential
E/E0
ϕ/ϕ0
0
0.2
0.4
0.6
0.8
1.0
0.6
0.3
0.0
-0.3
-0.6
-0.9
-1.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
X/λD


Figure 1

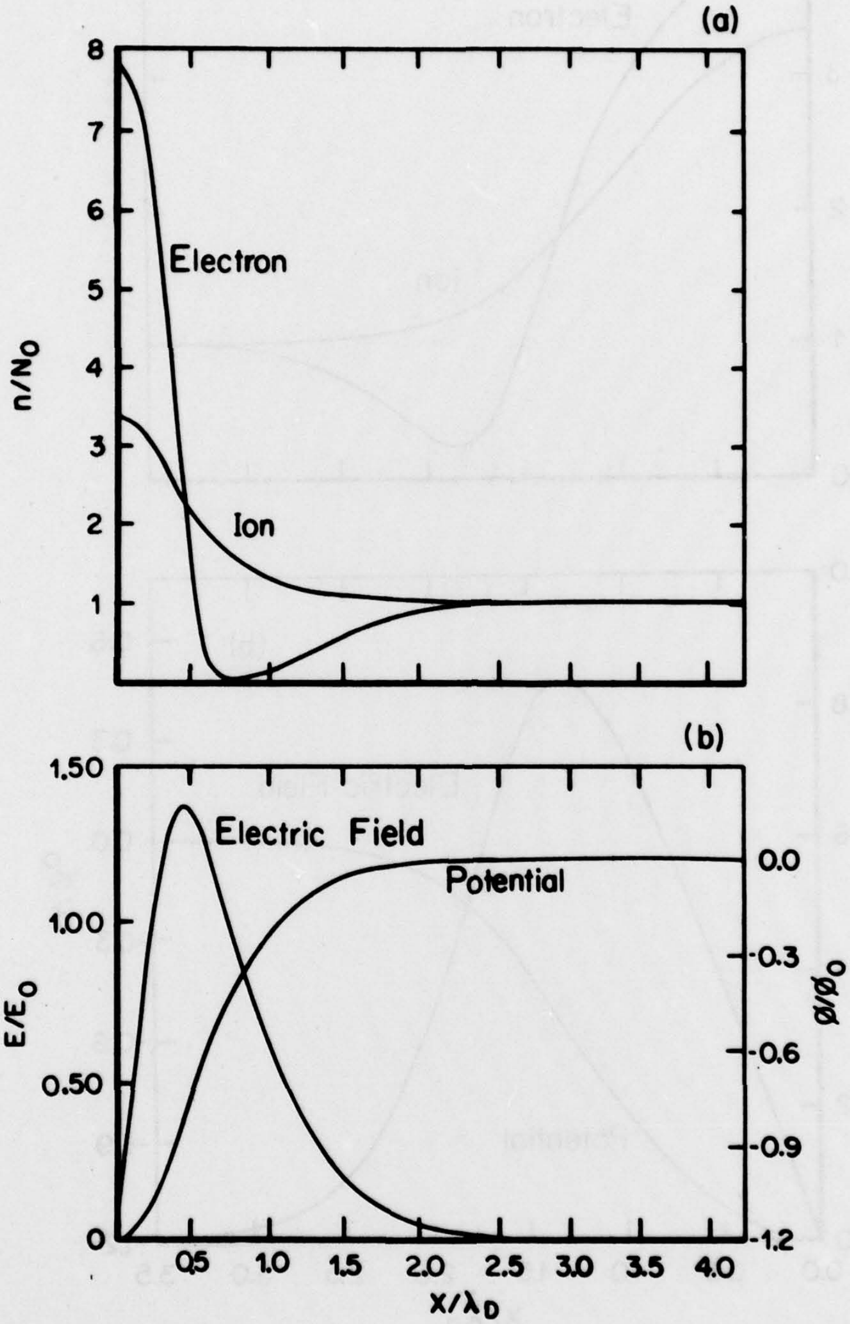
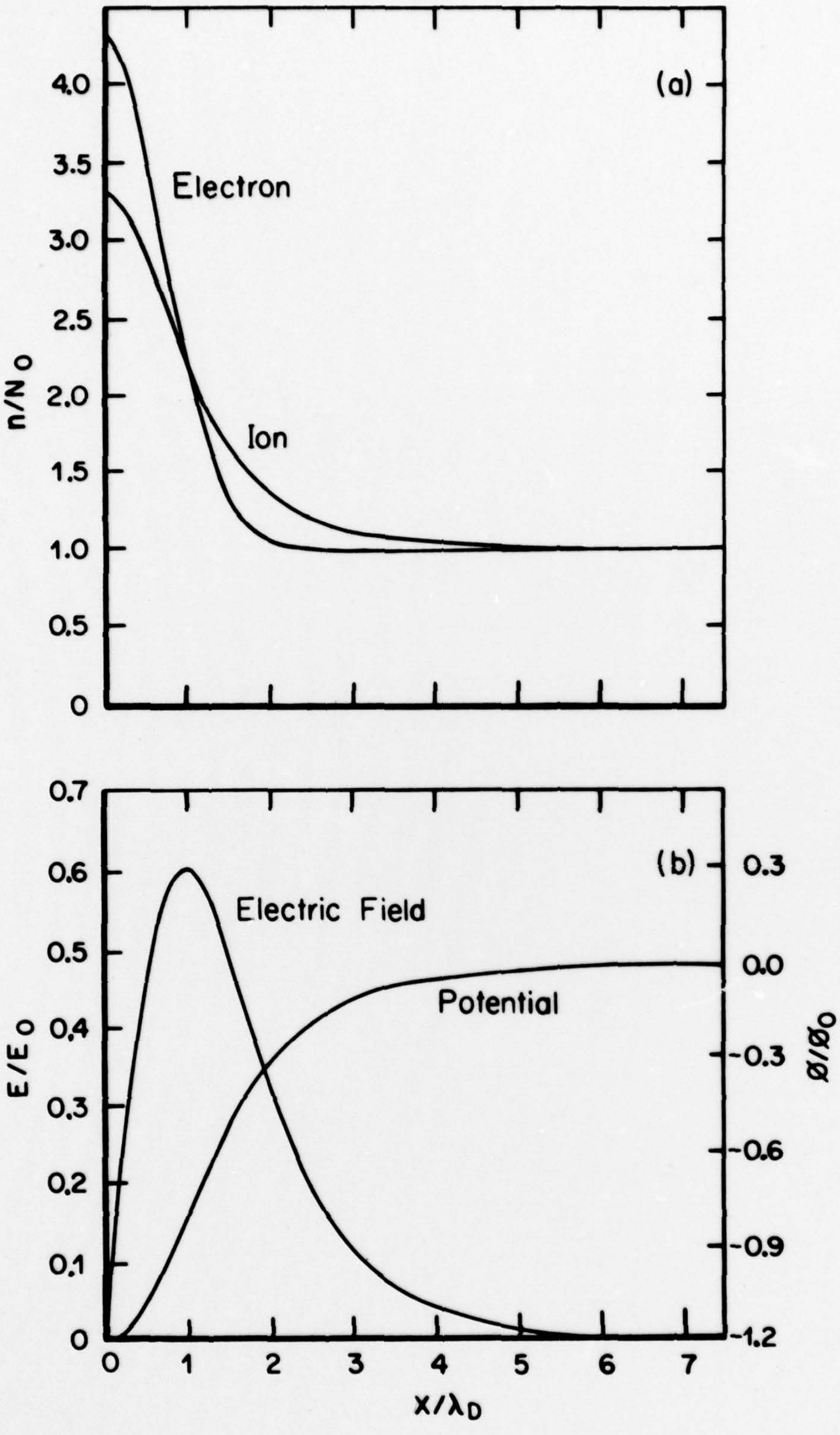

(a)
8
7
6
5
4
3
2
1
n/N0
Electron
Ion
(b)
1.50
1.00
0.50
0
E/E0
Electric Field
Potential
0.0
-0.3
-0.6
-0.9
-1.2
Ø/Ø0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
X/λD

Figure 2


(a)
Electron
Ion
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2.0
1.5
1.0
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0
(b)
Electric Field
Potential
E/E0
Ø/Ø0
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.3
0.0
-0.3
-0.6
-0.9
-1.2
0
1
2
3
4
5
6
7
X/λD

Figure 3